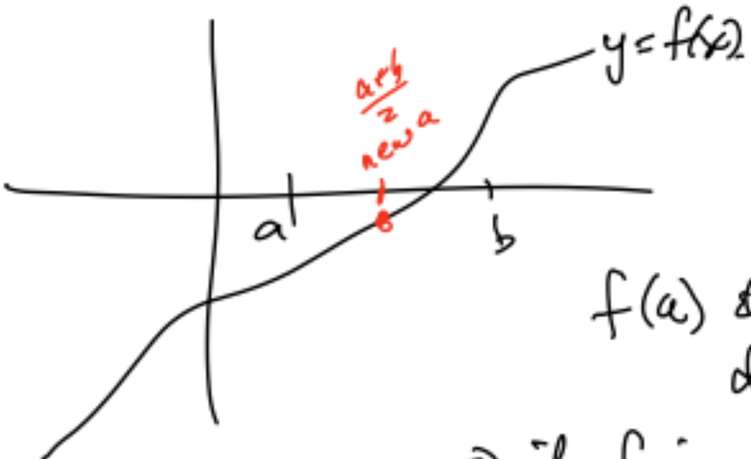
Solving Equations Numerically.

→ Equivalent to solving $f(x)=0$ for a given fcn. (by moving all to the left side.)

① Method of Bisection



$f(a)$ & $f(b)$ are different signs

⇒ if f is continuous, there must be a root (zero) between a & b ⇝ a number x s.t. $f(x)=0$.

↓

value $f\left(\frac{a+b}{2}\right)$ ← if $f\left(\frac{a+b}{2}\right)=0$, $\frac{a+b}{2}$ = a solution.

If not, replace one of a & b with $\frac{a+b}{2}$ → interval is half the size

Choose either a or b depending on if f(a) or f(b) has the same sign as $f\left(\frac{a+b}{2}\right)$.)

→ Keep doing this until the interval is very small, so you have a good approx. of the root.

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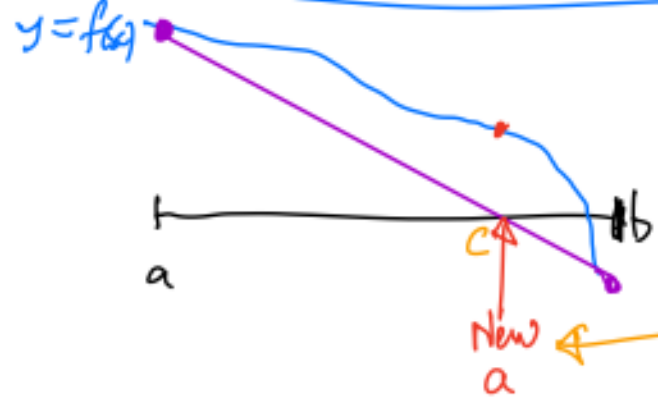
② Method of False Position

Regula Falsi



again, look at f(c), replace either a or b with c (pick the one with the same sign of the function.)

Formula for what c is:

Eqn of line connecting $(a, f(a))$, $(b, f(b))$

$y - y_1 = m(x - x_1)$

$$y - f(a) = \left(\frac{f(b)-f(a)}{b-a}\right)(x-a)$$

Where does it hit the x-axis? at $(c,0)$.

$$0 - f(a) = \left(\frac{f(b)-f(a)}{b-a}\right)(c-a)$$

$\neq 0$

$$\Rightarrow -\frac{f(a)(b-a)}{f(b)-f(a)} = c-a$$

$$\Rightarrow c = a - \frac{f(a)(b-a)}{f(b)-f(a)}$$

$$= \frac{a(f(b)-f(a))}{f(b)-f(a)} + \frac{-f(a)(b-a)}{f(b)-f(a)}$$

$$\Rightarrow c = \frac{af(b) - \cancel{af(a)} - bf(a) + \cancel{af(a)}}{f(b)-f(a)}$$

$$\Rightarrow \boxed{c = \frac{af(b) - bf(a)}{f(b)-f(a)}}$$

c = new side of interval.

Then continue as before.

Let's experiment with these methods on sagemath.

③ After experimentation: new suggested algorithm:

Secant Method

$[a, b]$

$$c = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)}$$

Next guess:

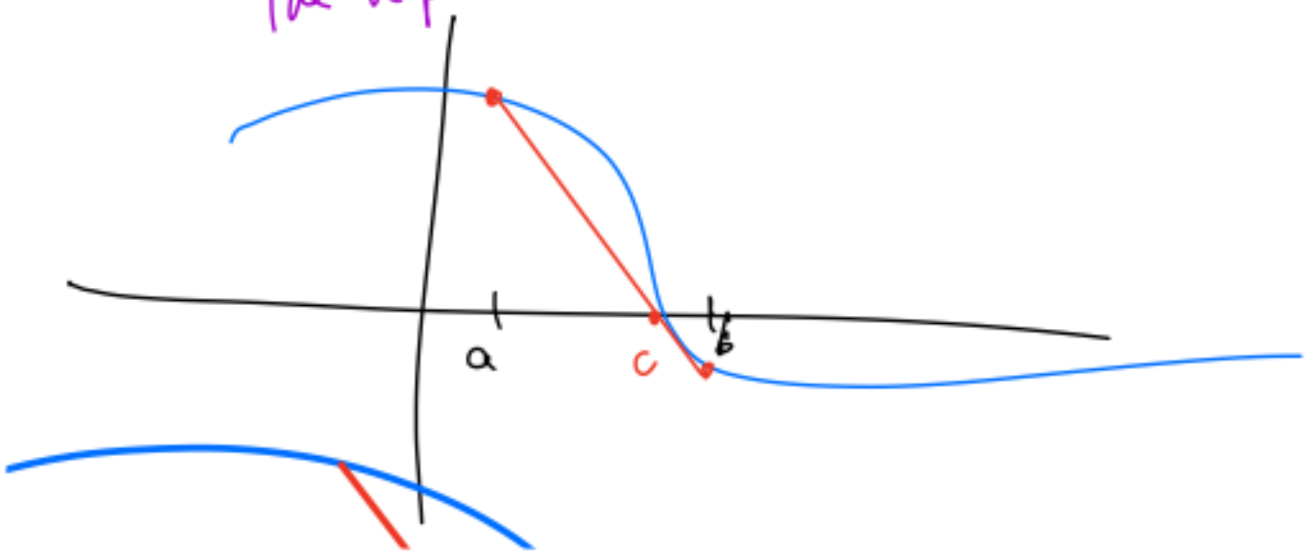
$$d = \frac{b\,f(c) - c\,f(b)}{f(c) - f(b)}$$

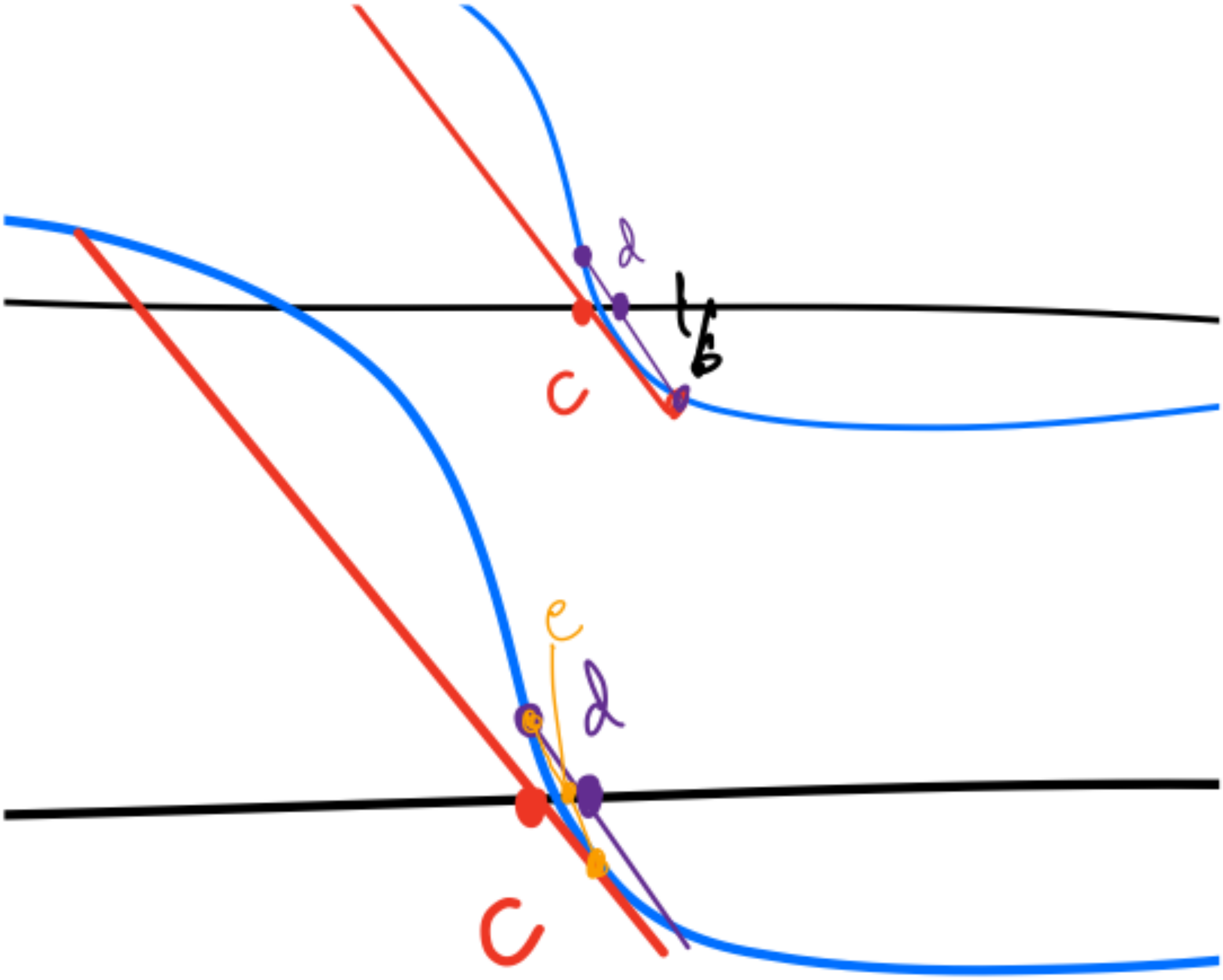
Next guess:

$$e = \frac{c\,f(d) - d\,f(c)}{f(d) - f(c)}.$$

$\vdots$

The hope: this gets closer & closer.

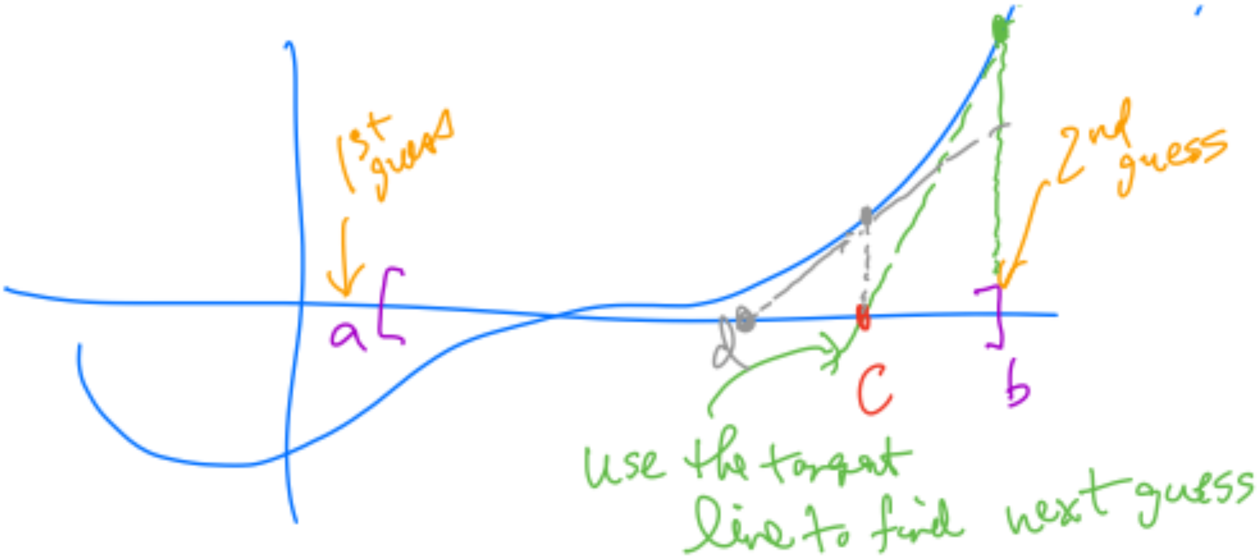


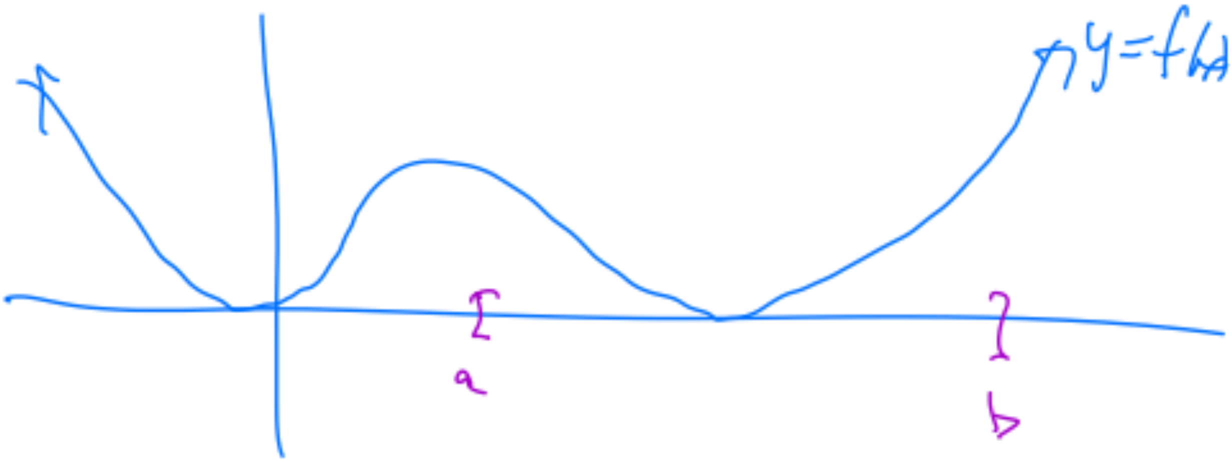


Back to Sagemath -

Newton-Raphson Method.

$y = f(x)$



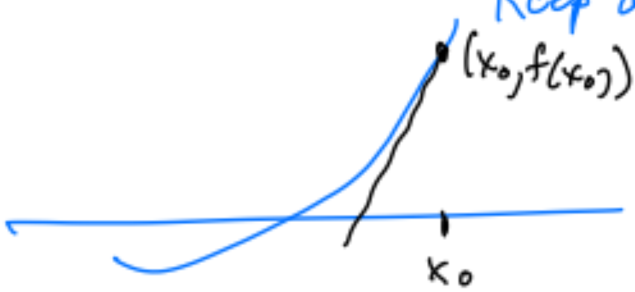


① Choose initial guess. x_0

② find tangent line @ $(x_0, f(x_0))$

→ let x_1 = pt where the tangent line hits the x-axis.

Keep on going (x_1 is the next guess).



$$y - y_0 = m(x - x_0)$$
$$y - f(x_0) = f'(x_0)(x - x_0)$$

at x-axis, $y=0$, $x_1=x$

$$0 - f(x_0) = f'(x_0)(x_1 - x_0)$$

$$\frac{-f(x_0)}{f'(x_0)} = x_1 - x_0$$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Sequence $\boxed{x_{j+1} = x_j - \frac{f(x_j)}{f'(x_j)}}$

$\hookrightarrow$ should converge to a root of $f(x)$.

Sagemath !